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PROBING NUCLEON STRANGENESS WITH
PARITY-VIOLATING ELECTRON SCATTERING¹

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ABSTRACT

The electromagnetic and weak neutral currents display different sensitivities to various strange quark matrix elements of the nucleon. Measurements of the parity-violating electron-proton and electron-nucleus asymmetries at intermediate energies could allow one to extract limits on the nucleon's strange-quark form factors. The prospects for deriving such constraints from present and approved experiments at MIT-Bates and CEBAF, as well as the implications of such measurements for models of nucleon strangeness, are discussed.

There has been considerable interest recently in the strange-quark content of the nucleon. In this talk, I will discuss how low- and intermediate-energy semi-leptonic scattering experiments might provide new information on the nucleon's strange-quark vector and axial vector current matrix elements. I will focus primarily on present and planned parity-violating (PV) electron scattering experiments at MIT-Bates and CEBAF. Although I can focus on only a few cases here, one may find a more detailed and comprehensive discussion elsewhere^{1, 2, 3}.

Neutrino and PV electron scattering experiments with nuclear targets probe the nucleon's weak neutral current (NC). In such processes, the lepton probe interacts with a hadronic target via the exchange of a Z^0 . At energies significantly below M_Z , the resulting interaction is a contact interaction between the leptonic neutral current, ℓ_μ^{NC} , and the neutral current of the target, J_μ^{NC} . In the Standard Weinberg-Salaam electroweak theory, the neutral current of elementary particles (leptons and quarks) has the form

$$J_\mu^{NC} = J_\mu^{W^0} - 4Q_f \sin^2 \theta_w J_\mu^{EM} \quad , \quad (1)$$

where $J_\mu^{W^0}$ is the $T_3 = 0$ partner of the charge-changing weak currents arising in semileptonic decays, J_μ^{EM} is the electromagnetic (EM) current of the elementary fermion, and Q_f is the fermion's electric charge. The Weinberg angle is fixed by the other parameters entering the Standard Model: $\sin^2 \theta_w \equiv \sin^2 \theta_w(\alpha, G_\mu, M_Z; m_t, M_H)$, where the fine structure constant, α , Fermi constant, G_μ , and Z^0 mass, M_Z are known very precisely. The top-quark mass, m_t , and Higgs mass, M_H , are constrained but not yet determined, and lack of knowledge in these parameters introduces some uncertainty into the Standard Model predictions for $\sin^2 \theta_w$.

The neutral current of the nucleon is simply the matrix element of an operator defined as a sum of the individual quark currents:

$$\hat{J}_\mu^{NC} = \sum_q J_\mu^{NC}(q) = \hat{V}_\mu^{NC} + \hat{A}_\mu^{NC} \quad , \quad (2)$$

where \hat{V}_μ^{NC} and \hat{A}_μ^{NC} are the vector and axial vector components, respectively, of the NC. If one "integrates out" the c , b , and t -quark currents, one may write \hat{J}_μ^{NC} in the following convenient form¹:

$$\hat{V}_\mu^{NC} = \xi_V^{T=1} J_\mu^{EM}(T=1) + \sqrt{3} \xi_V^{T=0} J_\mu^{EM}(T=0) + \xi_V^{(0)} \bar{s} \gamma_\mu s \quad (3a)$$

$$\hat{A}_\mu^{NC} = \xi_A^{T=1} A_\mu^{(3)} + \xi_A^{T=0} A_\mu^{(8)} + \xi_A^{(0)} \bar{s} \gamma_\mu \gamma_5 s \quad , \quad (3b)$$

where the $J_\mu^{EM}(T)$ are the isovector and isoscalar electromagnetic (EM) currents, the $A_\mu^{(3,8)}$ are the flavor-neutral SU(3) octet axial currents, and the $\xi_{V,A}$ are couplings determined by the underlying electroweak gauge theory and are renormalized to account for process-dependent radiative corrections. The effect of c , b , and t -quark matrix elements may be included as an additional renormalization of the $\xi_{V,A}$. The relevant corrections, $\Delta_{V,A}$, have been estimated by Kaplan and Manohar⁴ to be $\Delta_V \sim 10^{-4}$ and $\Delta_A \sim 10^{-2}$. For the purpose of this talk, I will assume the ξ_V to be known. The ξ_A , on the other hand, are theoretically uncertain in the case of charged lepton probes, once one accounts for the presence of higher-order electroweak processes.¹ The decomposition of Eq. (3) is convenient since it allows one to write the NC in terms of currents whose matrix elements may be measured in other processes plus an explicit strange-quark current remainder. Note that although the isoscalar EM and axial octet currents also contain s -quark terms, they cannot be experimentally separated from the u - and d -quark contributions to the same currents. Thus, because of the additional, explicit dependence on the s -quark currents in Eq. (3), the NC becomes a “meter” with which to measure hadronic matrix elements of $\bar{s}\gamma_\mu s$ and $\bar{s}\gamma_\mu\gamma_5 s$.

In the case of the nucleon, Lorentz covariance, together with parity and time-reversal invariance, require matrix elements of the operator (2) to have the form

$$\langle N(p') | \hat{V}_\mu^{NC}(0) | N(p) \rangle = \bar{u}(p') \left[\tilde{F}_1 \gamma_\mu + i \frac{\tilde{F}_2}{2m_N} \sigma_{\mu\nu} Q^\nu \right] u(p) \quad (4a)$$

$$\langle N(p') | \hat{A}_\mu^{NC}(0) | N(p) \rangle = \bar{u}(p') \left[\tilde{G}_A \gamma_\mu + \frac{\tilde{G}_P}{m_N} Q_\mu \right] \gamma_5 u(p) , \quad (4b)$$

where $Q = p' - p$ is the four-momentum transfer to the nucleon. The induced pseudo-scalar form factor, \tilde{G}_P , is not observable in the experiments of interest here, so I will not discuss it in detail. It is conventional in nuclear physics to work with Sachs form factors, defined in terms of the Dirac and Pauli form factors as

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \tau F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) , \end{aligned} \quad (5)$$

where $\tau = -Q^2/4m_N$, $Q^2 = \omega^2 - q^2$, and $q = |\vec{q}|$ with ω and \vec{q} being the energy and three-momentum transfer, respectively, to the nucleus and m_N denoting the nucleon mass (we ignore the n - p mass difference in this analysis). Following the decomposition of Eq. (3) one may write the NC form factors as

$$\tilde{G}_{(a)}(Q^2) = \xi_V^{T=1} G_{(a)}^{T=1} \tau_3 + \sqrt{3} \xi_V^{T=0} G_{(a)}^{T=0} + \xi_V^{(0)} G_{(a)}^{(s)} \quad a = E, M \quad (6a)$$

$$\tilde{G}_{(a)}(Q^2) = \xi_A^{T=1} G_{(a)}^{(3)} \tau_3 + \xi_A^{T=0} G_{(a)}^{(8)} + \xi_A^{(0)} G_{(a)}^{(s)} \quad a = A, P , \quad (6b)$$

where the $G_{(a)}^{T=0,1}$ denote the isoscalar and isovector electromagnetic Sachs form factors of the nucleon, the $G_{(a)}^{(3,8)}$ are the SU(3) octet axial vector form factors, and $G_{E,M,A}^{(s)}$ are the vector and axial vector strange-quark form factors.

From Eqs. (6), we observe that quite a bit is already known about the NC form factors. In particular, assuming the nucleon to be an eigenstate of isospin allows one to write

$$\begin{aligned} G_{(a)}^{T=1} &= \frac{1}{2}[G_{(a)}^p - G_{(a)}^n] \\ G_{(a)}^{T=0} &= \frac{1}{2}[G_{(a)}^p + G_{(a)}^n] \end{aligned} \quad a = E, M \quad (7)$$

Note that Eqs. (6) and (7) imply that the vector NC form factors of the proton depend on the electromagnetic (EM) form factors of the neutron. This result follows from the fact that the isoscalar and isovector form factors enter the NC with different weightings than in the case of the EM current. All of the form factors in Eqs. (6) have been determined from parity-conserving (PC) electron scattering. The proton EM form factors are known to rather high precision, whereas the bounds on the neutron form factors are less stringent. In particular, data recently taken at MIT-Bates may allow a 3 - 5% determination of G_M^n at low- $|Q^2|$, while G_E^n may be determined to roughly 50% precision. Further improvements in these constraints will be one of the aims of the experimental program at CEBAF.

In the case of the axial vector form factor, the quantity $G_A^{(3)}$ entering Eq. (6b) may be determined from neutron β -decay using isospin symmetry, so that its value at low momentum transfer is known quite precisely. The value of $G_A^{(8)}(0)$ may be obtained from hyperon semi-leptonic decays by employing an SU(3) parameterization of these decays. There is evidence that the SU(3) symmetry of the axial currents may be badly broken, with the result that the SU(3) value for $G_A^{(8)}(0)$ may be too large by a factor of two or more⁵. Naïvely, this symmetry breaking should not significantly affect our interpretation of \tilde{G}_A , since its coefficient $\xi_A^{T=0}$ in Eq. (6b) vanishes at tree-level in the Standard Model. However, radiative corrections render $\xi_A^{T=0}$ non-zero and, in the case of electron scattering, the contribution of this term to \tilde{G}_A is non-negligible.

The Q^2 -dependence of the above-mentioned form factors is fit rather well with a dipole form, $(1 + \lambda\tau)^{-2}$, with $\lambda \approx 5$ for the vector current form factors and $\lambda \approx 3.3$ for the axial vector form factors. An exception occurs in the case of G_E^n , which carries an extra Q^2 dependence as needed to reproduce the expected asymptotic behavior: $G(Q^2) \sim 1/Q^4$ as $Q^2 \rightarrow \infty$. In the Galster parameterization of G_E^n , this additional Q^2 -dependence has the form $(1 + \lambda_n\tau)^{-1}$, with $\lambda_n = 5.6$.

The remaining quantities appearing in Eq. (6) are form factors associated with the vector and axial vector strange-quark currents, $\bar{s}\gamma_\mu s$ and $\bar{s}\gamma_\mu\gamma_5 s$, respectively. Much less is known about them at present than in the case of the other form factors, and significant attention is being given to their determination at present. The only one of these form factors for which there exists published experimental constraints is $G_A^{(s)}(Q^2)$. Data from the Brookhaven $\nu p/\bar{\nu} p$ experiment and the EMC $\bar{\mu} p$ experiment indicates $|G_A^{(s)}(0)| \lesssim 0.19$. There exists considerable uncertainty in these results for a number of reasons. If, in fact, the value of $G_A^{(8)}(0)$ is a factor of two or more smaller than its SU(3) value, the value of $|G_A^{(s)}(0)|$ extracted from the EMC results would decrease by at least a factor of three. In the case of the BNL result, there exists a significant correlation between $G_A^{(s)}(0)$ and the value of the dipole mass used in parameterizing the Q^2 -dependence of the axial vector form factors. Clearly, a better determination of $G_A^{(s)}$ is of interest. The present constraints, however, are quite suggestive of significant strangeness in the proton, especially in light of analyses of $\Sigma_{\pi N}$, which suggests that $\langle p|\bar{s}s|p\rangle/\langle p|\bar{u}u + \bar{d}d + \bar{s}s|p\rangle$ is on the order of 10 - 20%.

No experimental information has been published on the vector current strangeness form factors, $G_E^{(s)}$ and $G_M^{(s)}$. The only rigorous theoretical constraint is that $G_E^{(s)}(0) \equiv 0$, since the nucleon has no net strangeness. All other theoretical statements about these form factors are model-dependent. In analogy with G_E^n , which also vanishes at the photon point, one may define a mean-square “strangeness radius”, which characterizes the low- $|Q^2|$ behavior of $G_E^{(s)}$. For purposes of this talk, I will use a dimensionless version of this quantity, ρ_s , defined as

$$\rho_s \equiv \frac{dG_E^{(s)}(\tau)}{d\tau} \Big|_{\tau=0} . \quad (8)$$

Similarly, one may define a “strangeness magnetic moment”, μ_s , as the value of $G_M^{(s)}(0)$. Various theoretical predictions for ρ_s and μ_s appear in the literature⁵⁻⁸. The pole model prediction of Ref. [6], for example, gives $\mu_s = -0.31 \pm 0.009$ and $\rho_s = -2.12 \pm 1.0$. Kaon loop calculations predict a value for μ_s similar to the one made with the pole analysis, but give a strangeness radius an order of magnitude smaller and with the opposite sign. Thus, measurements of these quantities could point to the correct picture for understanding the role played by strange quarks in the low-energy structure of the nucleon. To get a feel for the scale of these predictions, note that one has for ρ_n , the dimensionless mean-square charge radius of the neutron, $\rho_n \approx 1.91$.

Now let us consider various experiments in which the nucleon’s NC form factors may be probed. The low-energy cross section for charged lepton scattering is dominated by the EM interaction. In order to see the contributions of the NC, one must “filter out” the purely EM cross section by measuring a parity-violating (PV) observable. Specifically, one measures the left-right asymmetry, $A_{LR} = (N_+ - N_-)/(N_+ + N_-)$, where N_+ (N_-) is the number of counts detected when the incident electrons are polarized parallel (anti-parallel) to their momenta. This asymmetry is dominated by the interference of the parity-conserving EM and parity-violating NC amplitudes: $A_{LR} \sim 2M_{EM}^* M_{NC}^{PV}/|M_{EM}|^2$. Thus, A_{LR} is essentially governed by the ratio of the PV NC amplitude to the PC EM amplitude. This feature may be exploited to great advantage, as illustrated below.

An experiment is presently running at MIT-Bates aimed at constraining the low- $|Q^2|$ behavior of the strangeness magnetic form factor. The so-called “SAMPLE” experiment is measuring the PV left-right asymmetry in elastic $\bar{e}p$ scattering at backward angles. This asymmetry actually depends on several quantities:

$$A_{LR}(\bar{e}p) \Big|_{\theta \rightarrow 180^\circ} \sim a_o \tau \left[\xi_V^p + a G_M^n + b G_M^{(s)} + c G_A^p \right] , \quad (9)$$

where $a_o \approx 3 \times 10^{-4}$, $\xi_V^p = \frac{1}{2}[\xi_V^{T=1} + \xi_V^{T=0}]$, and a, b, c are kinematic coefficients. Of the quantities appearing in Eq. (9), ξ_V^p is the most precisely known, while $G_M^{(s)}$ and G_A^p are the most uncertain, the latter due to uncertainty in the radiative correction R_A^p associated with the axial vector form factor. At the SAMPLE kinematics, *uncertainties* in the various quantities are related as

$$\frac{\delta A_{LR}}{A_{LR}} \approx \frac{\delta R_A^p}{5} - \frac{\delta G_M^{(s)}}{3} - \frac{\delta G_M^n}{3} , \quad (10)$$

where the quantity on the left side of Eq. (10) is the fractional experimental uncertainty in the asymmetry. Note that even a “perfect” experiment would not allow one

to constrain the unknown quantities in the backward-angle $\bar{e}p$ asymmetry to arbitrary precision. The SAMPLE experiment expects to determine $\mu_s = G_M^{(s)}(0)$ to an error of $\approx \pm 0.2$ (see Ref. [9]) – a value having the same magnitude as the pole and loop predictions for μ_s .

Proposals have been conditionally approved at CEBAF for determinations of $G_E^{(s)}$ at low- and moderate- $|Q^2|$ with forward-angle measurements of $A_{LR}(\bar{e}p)$ (Refs. [10, 11]) and measurements of $A_{LR}(^4\text{He})$ (Refs. [11, 12]). In the former case, one has

$$A_{LR}(\bar{e}p)\Big|_{\theta \rightarrow 0^\circ} \approx a_o \tau \left[\xi_V^p - \left\{ G_E^n + G_E^{(s)} + \tau \mu_p (G_M^n + G_M^{(s)}) \right\} \right] , \quad (11)$$

where μ_p is the proton magnetic moment. Since both G_E^n and $G_E^{(s)}$ vanish as $\tau \rightarrow 0$, all of the terms inside the curly brackets are of $O(\tau)$. Note that in order to extract $G_E^{(s)}$ from a measurement of the above asymmetry, one requires knowledge of both the neutron EM form factors as well as of the strangeness magnetic form factor. Our knowledge of the former should improve over time, with the recent completion of the experiments at MIT-Bates and future experiments at CEBAF. A more serious issue is lack of knowledge in $G_M^{(s)}$, since it enters Eq. (11) with a coefficient of $\mu_p \lesssim 3$. In fact, in a “perfect” low- $|Q^2|$ measurement at $|Q^2| = 0.2 \text{ (GeV/c)}^2$ one has $\delta \rho_s \approx -3.7 \delta \mu_s$, assuming a reasonable form for the non-leading Q^2 -dependence. In a real experiment, of course, this relation will be modified by experimental error in the asymmetry. The bottom line is that forward-angle $\bar{e}p$ scattering would permit one to see vector current strangeness at some level, but would not allow one to constrain $G_E^{(s)}$ at a level needed to distinguish among theoretical models.

The latter statement leads one to consider PV elastic scattering from ^4He , for which the asymmetry is

$$A_{LR}(^4\text{He}) = -a_o \tau \left[\sqrt{3} \xi_V^{T=0} + \xi_V^{(0)} \frac{G_E^{(s)}}{G_E^{T=0}} \right] , \quad (12)$$

where $\sqrt{3} \xi_V^{T=0} = -4 \sin^2 \theta_w$ and $\xi_V^{(0)} = -1$ at tree level in the Standard Model. Note that in the absence of strangeness, this asymmetry is nominally independent of hadronic physics associated with the target and is determined solely by the electroweak coupling, $\sqrt{3} \xi_V^{T=0}$. The reason is that the target has $J = 0$ and, in the limit that states are eigenstate isospin, $T = 0$. Consequently, only one form factor enters the asymmetry, namely, an isoscalar electric form factor. In the absence of strange-quarks, this form factor is the same for the EM current and NC, up to the constant of proportionality, $\sqrt{3} \xi_V^{T=0}$. Strange-quarks introduce one correction to this result, as indicated by the presence of the second term in Eq. (12). In the one-body approximation, this term carries no dependence on the nuclear wavefunction, since the nuclear operators entering the matrix elements in the numerator and denominator are the same apart from the single nucleon form factor which enters as an overall factor. One also expects most of the nuclear correlations to modify the one-body matrix elements in the numerator and denominator in the same way, so that such effects should also cancel from the asymmetry. At low- $|Q^2|$, the strange-quark term grows linearly with $|Q^2|$, whereas the dependence of $\sqrt{3} \xi_V^{T=0}$ on $|Q^2|$, via radiative corrections, is more gentle. Thus, a separation of the two terms should be possible with a series of measurements. Alternately, one may take $\sqrt{3} \xi_V^{T=0}$ as a given input from the Standard Model, and determine $G_E^{(s)}$ at a given value of Q^2 with a single measurement.

In order to compare measurements of $A_{LR}(\bar{e}p)$ and $A_{LR}(^4\text{He})$ as ways to determine $G_E^{(s)}$, it is useful to parameterize the high- $|Q^2|$ behavior of $G_E^{(s)}$ as well as its low- $|Q^2|$ form. To that end, it is convenient to choose a Galster like parameterization, where the high- $|Q^2|$ behavior is characterized by a cut-off $\lambda_E^{(s)}$. Under this parameterization, $G_E^{(s)}$ displays the same asymptotic behavior as G_E^n , falling off like $1/Q^4$ as $Q^2 \rightarrow \infty$. It would not be unreasonable for this form factor to display an even more rapid fall off with large momentum transfer. Nevertheless, for purposes of comparing different possible experiments, the Galster-like form is sufficient. Taking both ρ_s and $\lambda_E^{(s)}$ as parameters to be constrained by experiment, one may then ask what constraints in $(\lambda_E^{(s)}, \rho_s)$ -space different measurements would impose. As discussed in Refs. [1-3] (especially, *e.g.* Fig. 4 of Ref. [2]), it appears from such analysis that a program of PV elastic electron scattering from ^4He has the potential to constrain $G_E^{(s)}$ significantly more tightly than a program with a proton target alone. The reason is essentially the relative simplicity of the ^4He asymmetry which, in contrast to $A_{LR}(\bar{e}p)$, contains only one poorly-known form factor.

There exist a variety of other possible PV electron scattering experiments using $A > 1$ targets as well as neutrino scattering measurements which could complement elastic scattering from the proton and ^4He (see Refs. [1-3] and references therein). Indeed, it appears that a program of complementary, low- and intermediate-energy measurements at CEBAF, MIT-Bates, LAMPF and other facilities has a unique potential for teaching us more about the strange-quark content of the nucleon.

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